2-52. A particle of mass m moving in one dimension has potential energy $U(x) = U_0[2(x/a)^2 - (x/a)^4]$, where U_0 and a are positive constants. **(a)** Find the force $F(x)$, which acts on the particle. **(b)** Sketch $U(x)$. Find the positions of stable and unstable equilibrium. **(c)** What is the angular frequency ω of oscillations about the point of stable equilibrium? **(d)** What is the minimum speed the particle must have at the origin to escape to infinity? **(e)** At $t = 0$ the particle is at the origin and its velocity is positive and equal in magnitude to the escape speed of part **(d)**. Find $x(t)$ and sketch the result.

2-53. Which of the following forces are conservative? If conservative, find the potential energy $U(\mathbf{r})$. **(a)** $F_x = ayz + bx + c$, $F_y = axz + bz$, $F_z = axy + by$. **(b)** $F_x = -ze^{-x}$, $F_y = \ln z$, $F_z = e^{-x} + y/z$. **(c)** $\mathbf{F} = \mathbf{e}_r a/r$ (a, b, c are constants).

4.2★★ Evaluate the work done

$$W = \int_O^P \mathbf{F}\cdot d\mathbf{r} = \int_O^P (F_x\,dx + F_y\,dy) \tag{4.100}$$

by the two-dimensional force $\mathbf{F} = (x^2, 2xy)$ along the three paths joining the origin to the point $P = (1, 1)$ as shown in Figure 4.24(a) and defined as follows: **(a)** This path goes along the x axis to $Q = (1, 0)$ and then straight up to P. (Divide the integral into two pieces, $\int_O^P = \int_O^Q + \int_Q^P$.) **(b)** On this path $y = x^2$, and you can replace the term dy in (4.100) by $dy = 2x\,dx$ and convert the whole integral into an integral over x. **(c)** This path is given parametrically as $x = t^3$, $y = t^2$. In this case rewrite x, y, dx, and dy in (4.100) in terms of t and dt, and convert the integral into an integral over t.

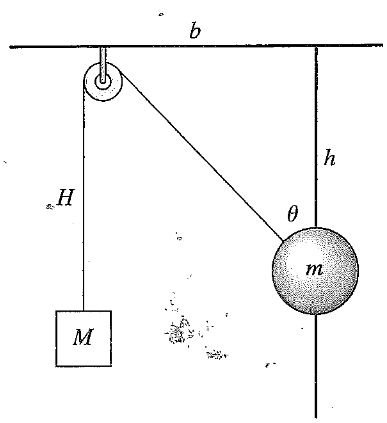
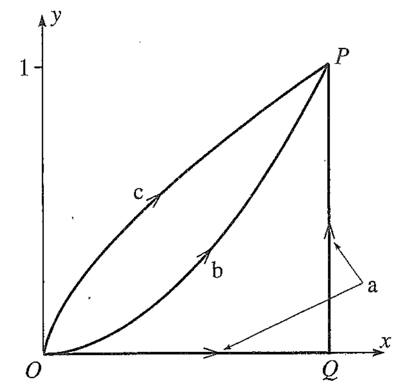


Figure 4.27 Problem 4.36



(a)

Figure 4.24

4.12★ Calculate the gradient ∇f of the following functions, $f(x, y, z)$: **(a)** $f = x^2 + z^3$. **(b)** $f = ky$, where k is a constant. **(c)** $f = r \equiv \sqrt{x^2 + y^2 + z^2}$. [*Hint:* Use the chain rule.] **(d)** $f = 1/r$.

4.36★★ A metal ball (mass m) with a hole through it is threaded on a frictionless vertical rod. A massless string (length l) attached to the ball runs over a massless, frictionless pulley and supports a block of mass M, as shown in Figure 4.27. The positions of the two masses can be specified by the one angle θ. **(a)** Write down the potential energy $U(\theta)$. (The PE is given easily in terms of the heights shown as h and H. Eliminate these two variables in favor of θ and the constants b and l. Assume that the pulley and ball have negligible size.) **(b)** By differentiating $U(\theta)$ find whether the system has an equilibrium position, and for what values of m and M equilibrium can occur. Discuss the stability of any equilibrium positions.